A METHOD FOR ACCOUNTING FOR

RISK IN LENDING

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Presented to
The Academic Faculty

by

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A METHOD FOR ACCOUNTING FOR

RISK IN LENDING

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SUMMARY

Many lending institutions, specifically furniture retailers, do not use scientific methods for determining their risk of payment defaults on loans to customers. The methods they do use can result in two problematic situations: they may not charge a high enough rate on a risky loan which results in unexpected losses; and they may charge too high a rate and lose business because of competition with lower rates.

In this thesis a simple model was developed for selecting the break-even interest rate to charge on risky furniture loans such that the expected present worth is zero, averaged over the population of borrowers. This model is a regression model based on loan parameters (contract length, markup, salvage value, face interest rate, and the lender's minimal attractive rate of return (MARR), but not individual borrowers' creditworthiness). In order to form such a model, data was necessary for the present worths of loans at various levels of the parameters in the model. This data would simply be in the form of a collection of payment histories on different loans with various parameters. Since data does not exist in the literature in this form, a detailed probabilistic model was first developed based on data from the mortgage-lending industry. Using this model, payment histories of furniture contracts were then simulated.

The final model recommended to a furniture credit manager is of the form:

$$f = .0149 - (0.000388 \times \text{length}) + (0.0385 \times \text{MARR} \times \text{length})$$

where f is the monthly nominal interest rate the credit manager should charge based on a given monthly nominal MARR and contract length. This equation on the average should result in a present worth of zero.

The adjusted coefficient of multiple determination for this regression was 94.6% and the p-value for the F-test for a regression relationship was 0.00000. It was concluded that this regression model extracted the data very well from the collection of payment histories which were simulated. A furniture credit manager can use this simple formula if he assumes that his borrowers behave as those in the mortgage industry. If the credit manager does not want to make this assumption, but would prefer to formulate his own simple model, he should follow the data collection process outlined in Chapter 3 and also follow the same steps in finding the above equation.

CHAPTER I

INTRODUCTION

Credit managers face the difficult task of determining the expected profit from consumer loans. This is particularly formidable because there is not as much information available about loan applicants as there is about other investment decisions such as bonds or the stock market [Chmura 95]. How then is a credit manager to determine if a loan at a specified interest rate is profitable or whether the interest rate should be raised to increase profitability and compensate for risk, or decreased to increase competitiveness?

Many lending institutions, specifically furniture retailers, do not use scientific methods for determining their risk of payment defaults on loans to customers. Techniques that exist for accounting for or compensating for risk vary in their scope from subjective increases in the lending rate to formulas involving various compensating factors which tell the lender the interest rate to charge. Many of these techniques will be cited in chapter II. None, however, seem to involve calculating a rate based directly on the risk of default. The non-scientific methods used by lenders to account for risk can result in two problematic situations: lenders may not charge a high enough rate on a risky

loan which results in unexpected losses; and lenders may charge too high a rate and lose business because of competition with lower rates.

Depending on the characteristics of the loan and of the contract, different probabilities of default occur at different times of the loan. For example, people are more likely to default on the first few payments of a contract than on those payments halfway through the contract. When calculating an expected present worth of a contract to determine its profitability at a specified interest rate, these probabilities of default should be taken into consideration. In addition to different probabilities of default, there are other factors which can influence the profitability of the contract. These factors include the following: 1) customers may not make their payments on time; 2) depending on the type of furniture sold to the customer and the wear and tear on it when a default occurs, there may be a cash flow from the sale of the repossessed furniture; and 3) there are costs associated with defaulted payments such as legal fees, administrative fees for sending late notices, and repossession fees such as appraisals of the furniture.

It would be beneficial for a furniture credit manager to have a simple model that could determine a contract's profitability, or perhaps a model that could calculate an interest rate that would allow him to break even. This break-even interest rate would be the interest rate such that the expected present worth for a loan with risk would be zero when computed at the enterprise's minimal attractive rate of return (MARR). This would be the lowest rate the credit manager could charge on a loan and still expect the contract to be profitable.

Knowing this interest rate can help a lender become more competitive in the lending industry.

In this thesis, a simple model will be developed for selecting the break-even interest rate for a risky furniture loan. This simple model will be a regression model based on several parameters. In order to form such a model, data is necessary to determine the present worths of loans at various levels of the parameters in the model. Ideally, this data would simply be a collection of payment histories on different loans with various parameters. As will be shown in chapter II, the only data that exists in the literature relating to payment histories is for the mortgage-lending industry. Thus, prior to developing the simple model that selects the break-even interest rate, a detailed probabilistic model will first be developed that simulates the payment histories of furniture contracts and then calculates the present worths of these contracts. The detailed probabilistic model will account for the different probabilities of default, the fees associated with defaulted contracts, any late payments, and for cash flows from repossessed furniture.

Raising or lowering interest rates based on the circumstances of the contract is called loan pricing. The loan pricing techniques used today may or may not include an analysis of risk, as will be shown in chapter II. In chapter II the current methods will be explored in the lending industry used to account for risk. The literature review will include lending in all areas, since there is little literature specifically on retail furniture credit management. In the third chapter a

detailed probabilistic model will be formulated that will simulate payment histories for numerous furniture contracts and will then calculate the present worths of the contracts. In chapter IV the design of the experiment will be discussed and several thousand different contracts will be simulated using the detailed model with various levels of its parameters. With the present worths that result from the simulations and their associated parameter values, a simple regression model will be developed that selects the interest rate to charge on a contract where the expected present worth is zero. Finally, in chapter V conclusions will be drawn from the simple model, appropriate recommendations made, and then ideas provided in this area for future work. Specifically, issues will be addressed concerning the two models developed, the detailed probabilistic model and the simple regression model. Recommendations will be made surrounding this research to improve the usefulness and accuracy of the simple model. Finally, future work will be recommended that can be accomplished in retail furniture loan pricing.

CHAPTER II

Literature Review

2.0 Overview

It is first necessary to study the previous work that has been done on risk in lending. Many studies have been done in developing different methods of accounting for risk; some methods are quite simple in their application while others are very complicated. The search began in the furniture industry and then was broadened to all lending industries. As expected, the banking industry's methods were much more sophisticated than those of other lending industries. In fact, very little evidence was found that industries other than banking use any methods at all. This is not to say that all banks use sophisticated methods, for most banks do not use scientific methods.

The first section of the chapter discusses the importance of accounting for risk in the lending industry and the importance of pricing techniques as a method for accounting for risk. The second section of the chapter discusses the various methods used for estimating risk: subjective evaluations of the borrower, the use of scoring models to categorize the borrower, and the determination of probabilities of default. In the third section, the applications of the management of risk to setting interest rates will be shown. Specifically, this section will

address the different pricing techniques used in the lending industry. Finally, in the fourth section of the chapter some applications of risk analysis to capital budgeting will be discussed.

2.1 The Need for Pricing Techniques

All who are in the lending industry know that they must have some way of accounting for risk. The profitability or survival of any lending business may depend on it. Several comments by leading managers and researchers in the lending industry pertain to this need for pricing techniques. John Haley, the vice president of Chase Manhattan Bank, stated that the ability of banks to generate needed capital depends on the successful achievement of pricing objectives [Haley 75]. He was referring to the possibility of banks going bankrupt if they did not develop sound pricing techniques. Haley further added that pricing may provide the key to future growth by improving on rates of return on assets. Timothy Finn and Joseph Frederick concluded from their research that superior lending means knowing good loans from bad loans, and that if a bank is to do one thing well, risk assessment should be it [Finn 92].

One would surmise that if so much could be lost on loans that default, then lenders would develop some formal way of coping with risk. What was found was quite to the contrary. In a survey conducted by *U.S. Banker's Magazine* in 1992, more than half of the respondents said that they did not use any credit decision products at all [Anonymous, 92]. Jon Kozlowski wrote that

most lending institutions employ less than scientific methods for pricing loans. Instead, lending rates are usually a function of the competitive environment [Kozlowski 96]. Lending rates that are set too low and are based on the competition can surely lead to disaster if a certain percentage of the loans default. Christine Chmura found proof of the lack of use of risk analysis techniques in pricing loans. She found that banks have been criticized for pricing loans to their best customers too high and loans to their riskiest customers too low [Chmura 95]. She described a survey that revealed that the pricing of loans is not consistent with the nature of the risk.

Further evidence of the lack of sophisticated pricing mechanisms can be found in the following research: Simon Hally concluded that nobody has figured out how to provide micro-credit cost-effectively in this country [Hally 96]. Micro-credit here refers to unsecured loans to very small businesses operated by people of limited means who are unable to qualify for loans from traditional institutions. Richard Johnson confirmed that further research is needed in the methodology for identifying risk premiums for credit managers [Johnson 90]. He stated that accounting for risk in pricing is not well-defined in the literature or in practice at many lending institutions. Finally, a senior vice-president at Wachovia Bank, Keith Lawder, insisted that lenders have historically done a "terrible" job in pricing loans [Goodwin 93].

Scott Aguais explained that as analytical techniques undergo rapid change relative to risk, the technology gap between where banks are now and

where they need to be is widening [Aguais 95]. He also noted that successful risk management requires accurate, flexible, consistent, and time synchronized information with a facility for collecting and storing significant quantities of historical data. Obviously, this can bring on significant costs for lending institutions, and this may explain why many do not partake in such analysis. Perhaps another reason for the lack of scientific methods in pricing is T. Pentikainen's finding that "rate-makers" are not always aware of the services of math and statistics [Pentikainen 88].

2.2 Managing the Credit Risk

Because of the potential losses that could result from lending to people who may default, one might think that it would be in the best interest of the lender to shy away from these situations. Of course, this idea is incorrect if the risk is managed correctly. This section will discuss methods currently used to account for risk: subjective evaluations and scoring models. It will also discuss some of the more current literature on managing risk with stochastic modeling.

2.2.1 Subjective Evaluations

The simplest analysis of the risk of a contract is a subjective evaluation of the borrower's financial situation. The subjective evaluation involves determining specific characteristics of the borrower and then placing the borrower into a category that represents a specific pricing range for the interest rate to be charged. There may also be a category in which the lender should not lend at

all. Surprisingly, a survey called the *1993 Survey of Credit Risk Management Practices* revealed that 72% of those lenders surveyed graded the risk of the borrower by judgment, only 2% by mathematical scoring, 25% by a combination of the two, and 1% by other means [Statistical Snapshot 94]. This survey shows that lenders still primarily use subjective means in determining the risk involved with loans. A reason for this may be the high cost of developing models to predict profitability. Walter Alexander noted that in order for a model to be truly predictive, it must be tailored to the specific requirements of the lending institution. This can lead to model-development costs of up to $100,000. This agrees with those findings of Aguais mentioned earlier [Alexander 89].

What characteristics do lenders use to categorize the borrower? In his book Consumers in Trouble, David Caplovitz discusses the following characteristics of the borrower: income, education, age, ethnic background, and location [Caplovitz 74]. To no one's surprise, a majority of defaulters come from the lower half of the income scale. Defaulters are normally young in age, usually less than forty, because younger families have greater consumer needs. It was a surprise to find that defaulters are well-educated, but it turns out that the young, who have a higher default rate, are better educated than the old. Caplovitz also explained that the primary reasons for default are loss of income, voluntary and involuntary over-extension, marital instability, and finally debtor irresponsibility. He also pointed out that the debtor is not always to blame for defaulted loans. Reasons for default that pertain to creditors are fraud, price

deception, defective merchandise, false promises, and payment misunderstandings.

2.2.2 Scoring Models

Some of the first actual research done on the analysis of risk in lending was done by "scoring" the borrower based on different characteristics. Scoring refers to assigning a number to an applicant based on a risk formula which depends on certain characteristics of the borrower. Many studies have been done to determine these characteristics. Edward Altman developed one of the first and most popular models, the z-score, which determines the risk in lending to firms based on data from general accounting reports (balance sheets and income statements) [Laudeman, 93]. The model Altman developed was based on several financial ratios that show the highest correlation to a firm's financial failure. His model was simple to use since it relied only on a few ratios to measure the overall financial position of the firm.

Jeffrey Marshall wrote about the success of another type of scoring, dynamic risk scoring [Marshall 92]. This method is tied to database management in that certain qualities of identified borrowers are maintained and continually updated in order to provide more accurate risk scores. Marshall noted that the better a bank's source of customer information, the more successful the bank will be. Many current software packages can provide output scores based on various risk profiles. Of the many scoring products that exist,

three of the most used are by TRW Information Systems, Trans Union, and Equifax, in that respective order [Anonymous, 94].

2.2.3 Determining the Probabilities of Default, the Expected Return, and the Expected Loss

The next level of analysis of risk is estimation of the probability of default. Delton Chesser developed a model to predict the probability that a commercial loan customer will not be able to comply with a loan agreement [Chesser 1974]:

$$P = 1/(1 + e^{-y})$$

where P is the probability that a commercial loan customer will not be able to comply, and y is estimated from a regression equation involving several financial ratios. Lawrence, Sanchez, and Smith developed the credit risk and management system (CRAMS) for mortgage-lending [Lawrence 96]. This system was designed to estimate under different economic scenarios the likelihood that each individual loan will terminate in default during each year of the loan's life.

Since there exist studies of default rates for corporate bonds, Chmura used these bond default rates to approximate commercial default rates and then developed this equation for expected return [Chmura 95]:

$$\mathrm{E}(R) = [(1 - d)(i - c) + d(IRR - c)]$$

where d is the default probability, i is the interest rate, c is the cost of funds, and *IRR* is the internal rate of return in the event of default. Asarnow and Edwards also used bond default studies as benchmarks in assessing the likelihood of corporate borrowers in defaulting. Their expected loss equation for a loan was [Asarnow 95]:

$$E(\text{Loss}) = P(\text{Default}) \times \text{Loss in the Event of Default (LIED)}$$

where LIED is determined by another equation representing characteristics of the contract in default.

2.2.4 Determining Distributions for Probabilities of Default

Up to recently, very little had been done in determining an actual probability distribution for defaults or late payments as a function of time for any lending institution. Gardner and Mills were among the first to collect data in this area. In a study of 710 loans, they found the following default rates by year [Gardner 89]:

Table 2-1 Initial Distribution Findings

Age of Loan at Time of Delinquency (years)	Number of Defaults	Percentage (Rate)
Age < 1	43	6.0
1 ≤ Age < 2	122	17.2
2 ≤ Age < 3	116	16.3
3 ≤ Age < 4	119	16.8
4 ≤ Age < 5	93	13.1
5 ≤ Age < 10	198	27.9
Age ≥ 10	19	2.7
TOTAL	**710**	**100.0**

As can be seen from this chart, the probability of default is highest in the first few years of the loan. This seems reasonable because one would think that once the borrower has made some payments, this behavior is predictive of making further payments.

The most recent study on credit risk involving default probabilities was by Lawrence, Sanchez, and Smith, who developed a comprehensive model for managing credit risk, a follow-up to the CRAMS model mentioned earlier [Lawrence 1996]. They structured this model as a Markov process to predict the probabilities that a loan will be in various states at annual intervals. They noted that at any time, a loan is classified as being in one of the following financial states: current, delinquent 30 to 89 days, delinquent 90+ days, defaulted, and

paid off. They then developed transition probability curves with data from 31,961 mortgages. These curves are shown in Figure 2-1 on the following page and reinforce Gardner's findings that the probability of default is greatest for a loan in the beginning. Note that in these curves, each plot has a y-axis range of [0,1] (transition probability) and an x-axis range of [0,1.2] (loan-to-value ratio). These curves will be addressed in more detail in chapter III. Given that a loan did default, Lawrence, Sanchez, and Smith then determined the magnitude of the loss. They modeled the percentage loss by the following linear regression equation:

$$P = 16.67 - 2\,(\text{loan \$150K to \$250K}) + 1.5\,(\text{loan \$250K to \$500K}) + 3.7\,(\text{loan over \$500K})$$

where P is the percentage loss on a loan and the loan-size terms are one if the statement is true and zero otherwise.

2.3 Pricing Techniques

Now that the research that has been performed in determining the risk of a loan has been seen, the applications of this research to pricing the loans will be shown. According to Chmura, there are three different pricing

Figure 2-1 Mortgage Default Probability Curves

mechanisms: risk-based pricing, portfolio theory pricing, and customer relationship pricing [Chmura 95]. Risk-based pricing refers to increasing or decreasing the interest rate based on the borrower's potential for defaulting. Portfolio theory pricing occurs when loans are priced relative to the risk the loan contributes to the lending institution's entire loan portfolio. Finally, customer relationship pricing occurs when the lender offers the borrower a lower than normal interest rate because the borrower is a good customer. As stated earlier, setting interest rates higher on risky loans can compensate for the higher risk; however, a higher interest rate also makes the lending institution less competitive in the lending market. First, some pricing techniques, which will be presented that do not specifically address risk, could be referred to as customer relationship pricing. Then some risk-based pricing techniques will be shown.

2.3.1 Techniques That Do Not Account for Risk

Numerous techniques for pricing loans exist when risk is not taken into consideration. Such pricing requires the bank to establish a minimum or target rate of return that must be earned on the loan and then compared to the calculated yield of the loan to see if the interest rate charged is high enough. One of the procedures discussed by Richard Johnson is called the *total funds used approach* [Johnson 90]. With this approach, the lender grants the customer credit for deposits at the bank's marginal cost of funds. Finn and Frederick noted that probably one of the simplest and most popular pricing strategies is *compensable-factor* pricing [Finn 92]. With this strategy borrowers

are rewarded with lower interest rates on loans for doing business with the bank in other areas. These other areas could include opening a checking account or keeping a minimum balance in a savings account. Similar to this pricing strategy is the *all-in-the-rate* pricing. John Haley described this strategy as giving the lender the option: some prefer keeping compensating balances for lower interest rates while others prefer paying a fee in lieu of maintaining balances [Haley 75]. In *ROE-based* pricing or *cost-plus* pricing, the price is established by determining the cost of the product, which includes a provision for loan losses, and adding on the required profit [Yang 91]. Gilbert Yang also mentioned in the same article that if the *ROE-based* price is lower than the market price, the bank will likely have an advantage over its competitors in obtaining loans [Yang 91].

2.3.2 Techniques That Account for Risk

One of the most recent models found for pricing that incorporated risk was by Thomas Stanley, Craig Roger, and John Lajaunie. They modeled the optimal loan price as follows [Lajaunie 95]:

$$\textit{Loan Price} = CF + AC + LOL + (RP \times ARI) \pm IBR - RWC$$

where *CF* is the cost of funds, *AC* are the administrative costs, *LOL* are the differential charges of the length of the loan, *RP* reflects the bank's attitude toward the minimum differential in credit risk premiums necessary to compensate for various costs, *ARI* is the actuarial risk index, which is the loan default

potential in the particular loan category (this comes from a table in their article), *IBR* is the borrower's credit score, and *RWC* is a subjective score of the bank-customer relationship.

Mike Casey and Ross Dickens modeled another pricing formula specifically for environmental risk [Casey 94]:

$$R = i + r$$

$$r = j \times d$$

where R is the rate charged, i is the normal interest rate charged with no risk , r is the environmental risk exposure factor, j is the risk factor category (one through four, one being that the company has no exposure to environmental risk), and d is the maximum potential environmental loss as a percent of the loan.

2.3.3 Pricing and Characteristics of Loans in the Furniture Industry

Several furniture retailers were visited in order to find out how they price loans. One company sells all loans immediately to a local bank. Another company noted that the rate they and many of the competitors charge is usually around the maximum rate allowed by the state law in which the store is in. Most states have ceilings of either 18% or 21% except for Arkansas, which has ceiling of 10.5%.

A peculiar pattern of default was found in the furniture industry; the highest levels of default are expected during the first few payments and during the final payments [Koonce 96]. A reason for this according to Ben Koonce, a senior credit manager for Haverty's Furniture, is because as people incur more debt with time, the most recent debts take the highest priority in paying, therefore leaving the older loans at a higher risk of default. Another reason may be that people have less fear of repossession because of the minimal value their furniture may have. Other facts learned from Koonce pertaining to furniture loans were as follows: the life of a furniture loan usually ranges from zero to 36 months, sometimes extending to 48 months, with the average being around 24 months; loan amounts usually average about $1200; and the markup average in the furniture industry is around 50% [Koonce 96].

2.4 Capital Budgeting Utilizing Risk Analysis

Related to accounting for risk in lending institutions is accounting for risk in capital budgeting. Recent surveys of capital budget techniques indicate that the simple and well-known procedure of adjusting the discount rate is the most popular method for accounting for risk [Hendricks 77]. James Hendricks noted that determining how much to increase the discount rate for different types of risk can be very difficult. He gave an alternative to using a different discount rate for each project using a different rate of discount for each class of investment such

as contractual obligation, on-going projects, new products and markets, and finally, research and development [Hendricks 77].

C.D. Zinn, W.G. Lesso, and B. Motazed commented on the use of Monte Carlo simulation models for evaluating risk in capital investment projects [Lesso 77]. They pointed out that an advantage of this simulation is that it provides management the convenience of using the probability distribution of values rather than single value estimates and that it also provides management with an estimate of the probability distribution of an investment's outcome. Young and Contreras showed how to calculate the expected present worth of cash flows where not only the amounts are uncertain but also the timing [Young 75].

Young's geometric risk model provides a direct interest-rate adjustment for a simple kind of risk. This simple risk is represented by the probability q that no further payments will be made and is the same each period. Independent of loan length, size, or any other factors, if i is the desired interest rate before risk, then the corresponding rate that should be charged to compensate exactly for geometric risk at the rate q is f in the following equation [Young 93]:

$$f = \frac{1+i}{1-q} - 1$$

Chapter III

Model Building

3.0 Overview

As shown in chapter II, there have been many studies on scoring borrowers, both *consumer* and *corporate*, and on determining *corporate* default probabilities. Very few studies, however, exists in the area of *consumer* default rates. Because of the lack of data for lending in the furniture industry, a probabilistic model was developed to describe the defaults of payments on loans to furniture retailers. As noted in the introduction, this model will be based on data from the mortgage industry and will result in a simulated collection of payment histories. This probabilistic model will be a stand-in for the data that is not available.

In the second section of this chapter a proof will be presented for the existence of a face interest rate f that makes a loan profitable at an interest rate of i. In the final section, a much simpler model, the geometric risk model, will be discussed but not investigated. This model could be used to predict the expected present worth of a series of loan repayments if the risk of default were constant with time [Young, 93].

3.1 Development of the Detailed Model of a Lending Contract

In developing the following representation of payment schedules, the only lending data available was used. This data was in the form of the mortgage-lending curves developed by Gardner, Lawrence, Sanchez, and Smith and was shown in Figure 2-1. It was initially assumed then that furniture and mortgage borrowers behave the same with respect to the loan-to-value (LV) ratio, the *x*-axis on all of the curves, which will be described in the next section. As in the mortgage-lending industry, in the furniture industry there are different states that the borrower's repayment behavior can occupy throughout the course of the loan. The detailed model developed here will be a Markov process and will employ state definitions similar to those of Lawrence, Sanchez, and Smith:

1. **Current** (with an outstanding balance and payments on schedule).
2. **Delinquent 30 to 89 days** (with an outstanding balance and payments overdue by 30 to 89 days).
3. **Delinquent 90+ days** (with an outstanding balance and payments overdue by 90 days or more).
4. **Defaulted** (with partial or complete charge-off incurred).
5. **Paid off** (with outstanding balance paid off at maturity or earlier).

These state definitions have the disadvantage that the times of transitions from one state to another do not exactly determine the timings or numbers of payments, so that a state trajectory for a customer does not unambiguously determine the present worth of the payments. However, the available data is in

this form, and, as will be seen, the ambiguities do not have a great effect on expected present worth.

As shown in Figure 3-1, a borrower starts off as seen below in the current state and can move to any one of the three states where the loan remains current, becomes late, or is paid in full. From the delinquent 30 to 89 days state and from the 90+ days state, the loan can move to any one of the five states. From the 90+ days state, the loan can return to the 30-89 days state if the borrower pays enough to become less behind, but not enough to become current or paid off.

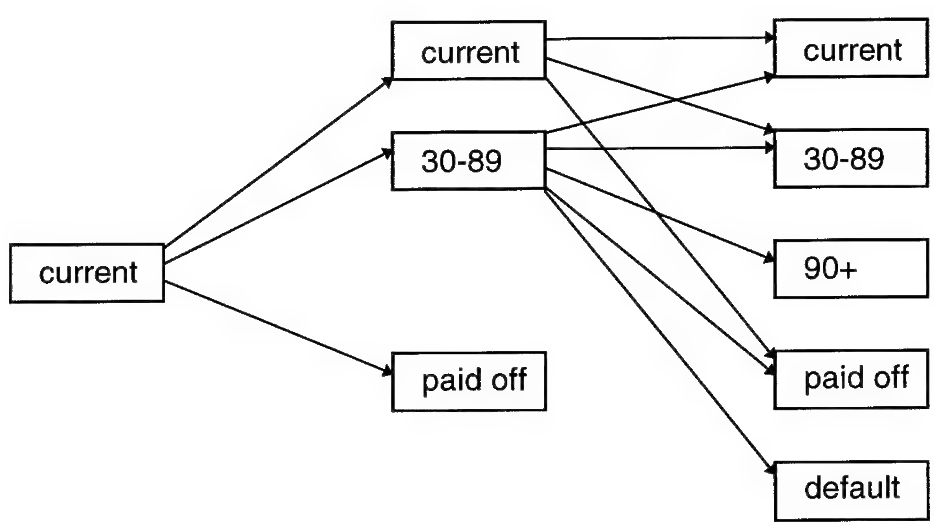


Figure 3-1 Stochastic Process Diagram

Once a loan has reached the paid-in-full or the default state, it never leaves these absorbing states.

Because of Koonce's comment mentioned in chapter II about higher probabilities of default at the beginning of and at the end of the life of the loan, a typical payment schedule was categorized into three groups: 1) the first fourth of the total number of months; 2) the next half of the total number of months; and 3) the final fourth of the total number of months. The intent was to assign for each of the three groups the same Markov process, i.e., with the same states, but with different probabilities. In effect then, three different probability matrices would represent the probability of transitioning to a new state for each of the payments.

In the transition probability curves shown in Figure 2-1, the probability of default is on the *y*-axis and a loan-to-value (LV) ratio is on the *x*-axis. As mentioned earlier, each plot has a y-axis range of [0,1] (transition probability) and an x-axis range of [0,1.2] (loan-to-value ratio). The numerator of this LV ratio is the remaining balance on the principal of the loan. The value amount in the denominator is the market value of the collateral; in the study in Figure 2-1 it is for homes. As time progresses on mortgage loans, the loan amount decreases and the value of the home usually increases. As a result, this LV ratio decreases as time progresses. This means that for any individual curve, time varies as one proceeds from right to left. Thus the probabilities of default are a function of time. As an example, the probability of default coming from the delinquent 90+ days state is highest when the LV ratio is highest, or the loan is at

its newest stage. This coincides with the findings of Gardner and Mills as noted in Table 1 [Gardner 89].

The transition probabilities from Figure 2-1 will be used in this study, but first the loan-to-value ratios for a particular furniture loan must be found. These ratios do not decrease with time as fast for furniture contracts as for mortgage contracts. Unlike property value, which usually appreciates in value with time, furniture value depreciates with time. So the rate at which the LV ratio decreases will depend on how fast the furniture depreciates, or how well the customer takes care of the furniture.

In order to determine the probabilities of moving from one state to the next for this study, the LV ratio points on the *x*-axis had to be found and then used to find the corresponding *y*-axis probabilities from the curves. To find these LV ratios, the loan balances and values for every month of the contract were found. The LV ratio for each month was then found by dividing the loan balance by the value for the particular month:

$$LV_t = \frac{balance_t}{value_t}$$

where t is the month of the contract. In order to determine the LV ratio for each month, it was assumed that typical furniture loans are around \$1200 and 24 months. With an average markup in the furniture industry of around 50%, the

actual value of the furniture at the time of the purchase would then only be $800. It was also assumed that furniture with an actual value of $800 would depreciate to $400, or 50% after 24 months. Of course depreciation rates vary for different pieces of furniture and depend on how much it is used, and depending on how well the owner takes care of it. This furniture then may depreciate from $800 to anywhere between $800 and zero. Assuming a geometric decline, a FORTRAN program was written to calculate the value of the furniture at every payment period or at every month. The program is shown in Appendix A and the depreciation schedule for this contract is shown in Figure 3-2 on the following page. The FORTRAN program utilizes the following equation for depreciation:

$$\text{depreciation rate} = \left(\frac{b(t)}{b(0)}\right)^{\frac{1}{t}} - 1$$

where $b(0)$ is the value of the furniture at time zero and $b(t)$ is the value of the furniture at time t. This depreciation schedule falls under the "value" column. These numbers are the denominators for the LV ratios for each respective month. Note that in order for the furniture to depreciate from $800 to $400 over 24 months, the rate of depreciation would be 2.85% per month.

The depreciation rate per month is 2.8468058846394D-02

The following represents the amortization and depreciation schedules for a 24 month $1200.00 loan at an interest rate of 0.18.

The markup is 0.50 %.
The salvage value is 0.50 %.

Note that at time *t*, *t* −1 payments have occurred.

	loan balance at time *t*	value at time *t*	loan/value at time *t*
t = 1	1200.0000000000	777.22555292288	1.5439533549652
t = 2	1158.0910776366	755.09945014535	1.5336934458284
t = 3	1115.5535214377	733.60323456374	1.5206496766623
t = 4	1072.3779018959	712.71897451227	1.5046293704047
t = 5	1028.5546480609	692.42924880491	1.4854292331471
t = 6	984.07404541841	672.71713220297	1.4628348206264
t = 7	938.92623373628	653.56618129644	1.4366199791332
t = 8	893.10120487891	634.96042078728	1.4065462596418
t = 9	846.58880058869	616.88433016318	1.3723623039100
t = 10	799.37871023411	599.32283075067	1.3338032012444
t = 11	751.46046852421	582.26127313687	1.2905898145617
t = 12	702.82345318866	565.68542494924	1.2424280743166
t = 13	653.45688262308	549.58145898324	1.1890082388005
t = 14	603.34981349902	533.93594166801	1.1300041192473
t = 15	552.49113833809	518.73582186040	1.0650722681087
t = 16	500.86958304975	503.96841995795	0.99385112879006
t = 17	448.47370443209	489.62141732186	0.91596014505484
t = 18	395.29188763516	475.68284600109	0.83099882822820
t = 19	341.31234358627	462.14107874891	0.73854578024153
t = 20	286.52310637666	448.98481932375	0.63815767047138
t = 21	230.91203060890	436.20309306610	0.52936816423240
t = 22	174.46678870462	423.78523774372	0.41168680068589
t = 23	117.17486817178	411.72089465740	0.28459781782337
t = 24	59.023568830946	400.00000000000	0.14755892207737

Figure 3-2 Depreciation Schedule

To determine the numerator of the LV ratios for every month or the balance remaining on the loan prior to the payment, a standard amortization formula was used [Young 93]:

$$B_n = A\,(1 - \beta^{N-n})/i$$

where B_n is the balance remaining after the nth payment, A is the monthly payment, i is the interest rate, N is the number of months the contract is for, and β equals $1/(1+i)$. It was assumed that the borrower repays in a series of equal monthly payments. The monthly payment A follows from the standard formula [Young 93].

$$A = P\,\frac{i}{1-\left(\dfrac{1}{1+i}\right)^{n}}$$

where A is the monthly payment, P is the amount the loan and equals \$1200, i is the monthly interest rate, and n is the number of payments and equals 24. A nominal interest rate of 18% was used which was found to be typical for retail furniture loans in most states. The corresponding monthly interest rate is i = 0.18/12 = 0.015 or 1.5% [Young 93]. The monthly payment came out to be A = \$59.91. The FORTRAN program shown in Appendix A also calculated the

balance at every time period through the life of the loan. The amount shown in the "balance" column is the balance owed *before* that month's payment.

Given the furniture values and the principal remaining on the loan for every month of the contract, the LV ratio for every month was determined. Once again, the program in Appendix A calculated these LV ratios for every month and is shown in the final column of Figure 3-2. Given these values, the curves in Figure 2-1 were used to develop the three probability matrices. The averages were taken of the LV ratios for the first six months, the next 12 months, and then the final six months. The average ratios for the first six months barely exceeded 1.2, which is the upper limit on the x-axis for the curves in Figure 2-1. Thus an LV ratio of 1.2 was used for the first six months. For the middle twelve months, the average was 1.18, and for the last six months the average LV ratio was 0.4583. Using these numbers, the three probability matrices were determined and are shown below.

1st 6 months

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} .75 & .04 & 0 & 0 & .21 \\ .32 & .16 & .24 & .20 & .08 \\ 0 & 0 & .02 & .98 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Middle 12 months

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} .72 & .04 & 0 & 0 & .23 \\ .35 & .18 & .21 & .17 & .09 \\ .005 & 0 & .02 & .97 & .005 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Last 6 months

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} .67 & .03 & 0 & 0 & .30 \\ .45 & .22 & .05 & .03 & .25 \\ .18 & .05 & .15 & .44 & .18 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Figure 3-3 Stochastic Process Matrices

These transition probabilities are for a 24 month contract, an interest rate of 18%, a markup on the furniture of 50%, and assuming the salvage value of the furniture at the time of default is 50% of the initial value. Note that all of these parameters will vary in the experimental design in chapter IV. Probability matrices were similarly developed for each of the simulations with different parameters.

The final aspect of this model to be discussed is the calculation of the present worth of a series of hoped-for cash flows. Given the probabilities, it is easy to run simulations of thousands of contracts in order to simulate their payment histories. Throughout the simulation of each individual contract, the present worth is maintained as a running total right up to the end of the simulation. The reason for this is as follows: if you run a simulation of a contract and find that it defaulted on the 10th payment and you did not continuously update the present worth after every payment, you would not know if the first nine occurred on time, or if one or more payments were late. Therefore, after every transition to the same or different state in the Markov process, the present worth must be calculated. It will be assume that all payments are received at 30 day increments. The cumulative present worth will be added in the fashion described in Table 3-1.

The formula column of this table represents that which is added to the cumulative present worth for a specific contract as this contract progresses

through the simulation. t represents the month the contract is currently in during the simulation and $\beta = 1/(1+i)$, i being the monthly discount rate at the minimum attractive rate of return (MARR). For example, if the contract remains

Table 3-1 Present Worth Formulas

Leaving State	Arriving State	Formula
1	1	$+ (59.91)\beta^t$
	2	$+ 0$
	3	zero probability
	4	zero probability
	5	+ balance(month) β^t
2	1	$+ 2(59.91)\beta^t$ (if 30 days late)
	1	$+ 3(59.91)\beta^t$ (if 60 days late)
	2	$+ 0$ (if 30 days late)
	2	$+ (59.91)\beta^t$ (if 60 days late)
	3	$+ 0$
	4	$+ (X - 400)\ \beta^t$ (if recoverable)
	4	$- 200\beta^t$ (if not recoverable)
	5	+ balance(month) β^t
3	1	+(time in state 3 + 3)*(59.91) β^t
	2	+ (time in state 3 + 1 or 2) $\times$ (59.91) β^t
	3	$+ 0$
	4	$+ (X - 400)\ \beta^t$ (if recoverable)
	4	$- 200\beta^t$ (if not recoverable)
	5	+ balance(month) β^t

in state 1, the current state, the payment calculated earlier of $59.91 will be discounted by the number of months *t*, which is the month the payment is made in and added to the cumulative present worth for that contract. If the borrower goes from being current to being delinquent, state 2, nothing is added to the cumulative present worth for that contract because no money is received. Note that there is a zero probability of moving from state 1 to states 3 or 4 because the contract must first become 30 days late, state 2. The final possibility of moving from state 1 is to state 5 where the balance of the loan is paid off and then discounted to the present.

If a contract moves from state 2 to state 1, two payments may be made if the contract was 30 days late, or three payments may be made if the contract was 60 days late; a contract is only in state 2 if it is 30–89 days late. The number of payments made to catch up, either two or three, are then discounted and added to the cumulative present worth for that contract. A similar scenario occurs when the contract remains in state 2. If the contract moves to state 3, no payment is received. If the contract moves from state 2 to state 4, i.e. it becomes defaulted, the furniture may or may not be recoverable. If it is recoverable, then the depreciated value of the furniture, *X*, is received minus legal and repossession fees of $400 and then is discounted. The repossessed furniture was assumed to be worth the amount of the depreciated value at the time of the default; a 50% chance of recovering the furniture was also assumed. Of course depending on how well the borrower takes care of the furniture, the

furniture may be worth more or less. If the furniture is not recoverable, then $200 was spent on the attempt at recovery and then this fee is discounted. The $200 and $400 are estimates [Koonce 96].

For contracts moving from state 3 to state 1, the number of payments discounted will be the number of months in state 3 plus three more payments that will make the contract current. If the contract moves from state 3 to state 2, the number of payments discounted will be the number of months in state 3 plus one or two payments. The reason for this is because a contract in state 2 can be either one or two payments late. A 50/50 chance was assumed in the simulation of being one payment late. A final assumption made during the simulation was that when a contract remained in state 3, no payment was made. In other words, it is not possible to go from 180 days late to 150 days late; once a contract is in state 3, it either becomes more late or it is paid enough so that it moves to state 2. Finally, states 4 and 5 are absorbing states and so are not depicted in the 'leaving state' column. After these simulation runs, the average present worth of the contracts was calculated.

The FORTRAN program which simulated the contracts and implemented the present worth calculations in Table 2 is shown in Appendix B. This program utilized a random number generator developed by Marse and Roberts and modified by Law and Kelton [Law, 91]. Law and Kelton tested this generator with much success and noted that the period is $2^{31} - 1$ [Law, 91]. The maximum

possible numbers that the program in Appendix B could use is 116,640,000, well below the period of the generator.

After performing some initial simulation runs with the probability matrices shown in Figure 3-3, the results were checked to see if they made sense in the furniture industry. Koonce pointed out that only 2-3% of loans are paid off early compared to the near 50% in the results which used the mortgage industry data [Koonce, 97]. The reason the mortgage industry data had such a high number of paid off early contracts is probably because with home mortgages, many of the loans are either refinanced or the homes are sold. Both of these factors may have counted in the data of Lawrence, Sanchez, and Smith as being paid off early. Obviously, this does not happen in the furniture industry.

Koonce also noted that there is a 3-5% chance of default on any given loan [Koonce 97]. Once again, the results of the initial simulation runs produced too many defaulted loans which meant that the probability of default is much higher in the mortgage industry than in the furniture industry. A final point learned from Koonce was that once a payment is late, there is around a 50/50 chance of that payment becoming current depending on how late the payment is [Koonce 97]. This means that if a contract is in states 2 or 3, there is a 50% chance of the contract becoming current during the next month or moving to state 1. Of course, the later a payment is, the lower the chance of the contract becoming current. This 50% probability is a lot higher than the probabilities of moving from states 2 or 3 to state 1 in the mortgage data as shown in Figure 3-3.

With these facts a new set of probability matrices was formulated that more accurately depicts the furniture industry loans. These matrices, which still use many of the basic assumptions and probabilities of those in Figure 3-3, are shown in the figure below. The primary changes are that there is a much lower chance of paying off the loan i.e., moving from any of the first three states to state 5, and a much higher probability of becoming current once late.

1st 6 months

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} .979 & .0195 & 0 & 0 & .0015 \\ .715 & .10 & .10 & .084 & .001 \\ .32 & .20 & .30 & .18 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Middle 12 months

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} .981 & .018 & 0 & 0 & .001 \\ .726 & .10 & .09 & .083 & .001 \\ .35 & .19 & .29 & .17 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Last 6 months

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} .979 & .0195 & 0 & 0 & .0015 \\ .715 & .10 & .10 & .084 & .001 \\ .32 & .20 & .30 & .18 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Figure 3-4 Revised Stochastic Process Matrices

Initial simulation runs with this data produced much more realistic results for the furniture industry. Thus the stochastic processes described in Figure 3-4 will be the best estimation of how loans are repaid in the furniture industry. There is no claim, of course, that this model is accurate for furniture loans. However, after having been calibrated to yield the appropriate default rates, early paid-off rates, and late-to-current rates, it does provide the basis for generating simulated payment histories that appear reasonable in furniture lending.

If one was able to collect data on the repayment of loans in the furniture industry, it should be collected in the following manner. For each loan a record should be kept on the timing of each payment. This record would intrinsically include: if and when payments were late; if and when these late payments became current; and if the contract defaulted, the timing of the default. Using this record, the present worths of each of the contracts can be calculated at a desired MARR, markup, salvage value, length, and at a particular interest rate charged. Note that all of the parameters pertain to the lender and the contract, none to the borrower. All characteristics of the borrower that might have an impact on the probability matrices have been excluded in this analysis because of the lack of data pertaining to these characteristics.

As will be explained in the design of the experiment in the next chapter, the five parameters mentioned in the preceding paragraph can be varied in order to formulate an accurate model for selecting the interest rate to charge where the present worth is zero. Because the variation of these parameters is discussed in chapter IV, it will suffice to say here that a lender could collect data on perhaps 100 loans to provide a sufficient database to form a good model.

Referring to David Caplovitz's findings discussed in Chapter II, there are five characteristics of a borrower that can predict the likelihood of default: income, education, age, ethnic background, and location [Caplovitz 74]. Even better data for this research would be if one could keep records on the contracts with the five varying parameters as discussed in the preceding paragraph, and

then also maintain for each of those borrowers the values of Caplovitz's five characteristics. In order that the data might cover the full range of possibilities of Caplovitz's traits, data would have to be kept on several hundred loans. These traits could become potential predictor variables to be added to the model if they were found to be significant.

3.2 Proof of the Existence of a Face Interest Rate f to Make a Loan Profitable at an Interest Rate of i

The following theorem will now be proven.

Theorem: If the probability of every payment is positive, then a face interest rate f exists such that the loan is profitable at an interest rate of i.

Proof:

Definitions:

- the MARR for a non-risky investment is represented by i and β equals $1/(1+i)$.

- standard loan is a series of cash flows $\{A_0, A_1, \ldots, A_n\}$ where $A_0 < 0$ and $A_t = A > 0$, $1 \le t \le n$, where A is the hoped for repayment amount.

- f is the substitute interest rate or the interest rate on the loan for a risky investment. Then the payment A required is

$$A = \frac{|A_o| f}{1 - \left(\frac{1}{1+f}\right)^n}$$

Now the expected present worth of the loan at a monthly nominal MARR of *i*, where π_t is the probability that the repayment of amount *A* due at time *t* will be paid at time *t*, otherwise never, is given by the following equation:

$$E(P) = -|A_o| + A(\pi_1\beta + \pi_2\beta^2 + \ldots + \pi_n\beta^n)$$

If $E(P) > 0$, this implies:

$$A > \frac{|A_o|}{\pi_1\beta + \pi_2\beta^2 + \ldots + \pi_n\beta^n}$$

Since all probabilities are greater than zero, the denominator of the above equation is also greater than zero, which implies that *A* is greater than some constant *C* where

$$C \equiv \frac{|A_o|}{\pi_1\beta + \pi_2\beta^2 + \ldots + \pi_n\beta^n}$$

This in turn implies that given $|Ao|$ and *n*, $\text{LIM A}_{f \to \infty} \dfrac{|A_o| f}{1 - \left(\frac{1}{1+f}\right)^n} = |A_o| f = \infty$.

Therefore, since A can be made arbitrarily large by setting f sufficiently large, $E(P)$ can be made positive. The loan can be made profitable by setting f high enough.

End of Proof.

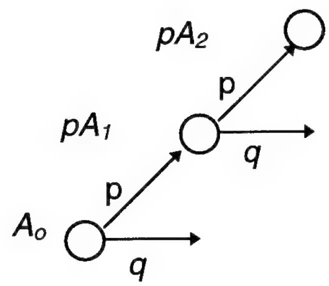
In Section 3.1, the goal was to determine the expected present worth of a series of hoped-for cash flows using the detailed model which was developed. This was done by performing many simulation runs using the probability matrices shown in Figure 3-4, then finding the average of these present worths. The next step is to calculate the new face interest rate f that will give a present worth of zero for these contracts involving risk.

3.3 Geometric Risk Model

A much simpler model than the stochastic process just developed for finding the expected present worth of a series of hoped-for cash flows is the geometric risk model [Young, 93]. Being a one-parameter model, the geometric risk model is not rich enough to provide a basis for the loan pricing model described in the preceding sections. However, it is of theoretical interest because it provides an average default rate over the life of the loan. This default rate is an integral of the time-varying default rates in the sense that a constant default rate q expresses the varying default rates independently of interest rates.

The geometric risk model contains a single parameter, q, which is the equivalent constant conditional probability of default for each period in the life of

the loan. Let $p = 1 - q$, where p is the conditional probability that default does not occur. When a payment occurs, q is the conditional probability that no further payment will occur. Consider the following series of hoped-for cash flows $\{A_t: t = 0,\ldots,n\}$, where there is a risk q at each time period that no further payments will occur:



Here q is constant. As mentioned earlier, this is not the case in reality, as default probabilities will vary with time.

Figure 3-5 Geometric Risk Model

The credit manager considers that the payment A_o is certain to occur (this is the negative cash flow; in the furniture-credit context it represents the delivery of furniture to the customer, less the down payment, if any), but that the payment at time $t = 1$ has a probability p of occurring so the expected value of this cash flow is pA_1. The conditional probability of the payment at $t = 2$ occurring, given that the payment at $t = 1$ did occur, is p^2. The expected payment at time t is p^tA and the expected present worth of a set of hoped-for payments under geometric risk is

$$E(P) = \sum_{t=0}^{n} p_t A_t \beta^t$$

where n is the number of payments and $\beta = \frac{1}{1+i}$, i is the interest rate charged, and A is the constant monthly payment [Young, 93]. Now if a loan L was found, justified by the $\{A\}$, $\{p^t\}$, and $\{\beta^t\}$ such that

$$L = E(P) = \sum_{t=0}^{n} p_t A_t \beta^t$$

a substitute interest rate f could then be found so that

$$\sum_{t=1}^{n} A_t \alpha^t = L = \sum_{t=1}^{n} A_t p_t \beta^t$$

where $\alpha = \frac{1}{1+f}$. This then becomes a rate of return calculation in solving for f.

It will now be shown that any expected present worth of a series of hoped-for cash flows using a model of the probabilities of receiving future cash flows can be found using the simple geometric risk model. Once the face interest rate f has been determined, the geometric risk parameter q can be calculated.

$$\sum_{t=1}^{n} A_t \alpha^t = \sum_{t=1}^{n} A_t p_t \, \beta^t$$

implies that $\alpha^t = p_t \beta^t$ for all t. This in turn implies that $p_t = p^t$ and $\alpha^t = p^t \beta^t$.

Finally, $p = \frac{\alpha}{\beta}$, where $p = 1 - q$, $\alpha = \frac{1}{1+f}$, and $\beta = \frac{1}{1+i}$. Knowing *f*, which was just determined, and *i*, the monthly nominal MARR, q can be solved for:

$$q = 1 - \frac{1+i}{1+f}$$

Recall that an expected present worth of zero implies that

$$|Ao| = A(p_1 \beta + p_2 \beta^2 + \ldots + p_n \beta^n)$$

This now equals A(P/A,f,n), where as shown above, $p\beta = \alpha = \frac{1}{1+f}$.

A calculation has now been shown that will determine a single value of q that represents the 75 different probabilities shown in Figure 3-4. Given a model that can predict q and also given a MARR, a lender can easily determine a face interest rate to charge that will result in an expected present worth of zero by

using $q = 1 - \frac{1+i}{1+f}$ and solving for f. This rate would represent the floor on how low of a rate the lender can charge and still be profitable.

The three quantities i, f, and q are related by the formula:

$$1 + i = (1 + f)(1 - q).$$

Any two of the three quantities could represent the overall relationship of risk to interest rate. Since i and f represent, respectively, MARR and the interest rate for a loan, they are more intuitively meaningful to a furniture lender than is q. Therefore, q will not be explicitly used in the loan pricing computations to follow.

CHAPTER IV

Loan Pricing Model Computation

4.0 Overview

The goal in this chapter is to develop a simple loan pricing model from a desired rate of return and from a loan's repayment behavior. This repayment behavior will be simulated by the detailed probabilistic model described in chapter III. In the first section of this chapter the experimental design is developed. A model of interest rates to be charged will be based on the following parameters: MARR, length of the loan, salvage value, markup, and present worth. The experiment will follow the logic flow chart in Figure 4-1 on the following page. A Monte Carlo simulation will yield the present worths of numerous contracts. The loan interest rate will be then be regressed against five parameters: present worth, length, MARR, markup, and salvage value. Once this regression equation is obtained, the interest rate to charge a borrower where the expected present worth is zero will be found by setting the present worth in the regression equation equal to zero and substituting the remaining parameters for a given loan.

If data was available for actual loans, the initial part of the experiment could be bypassed and the present worths from these contracts could

immediately be calculated. The final section of this chapter will evaluate and validate the regression model in selecting the *f*, the face interest rate to be charged.

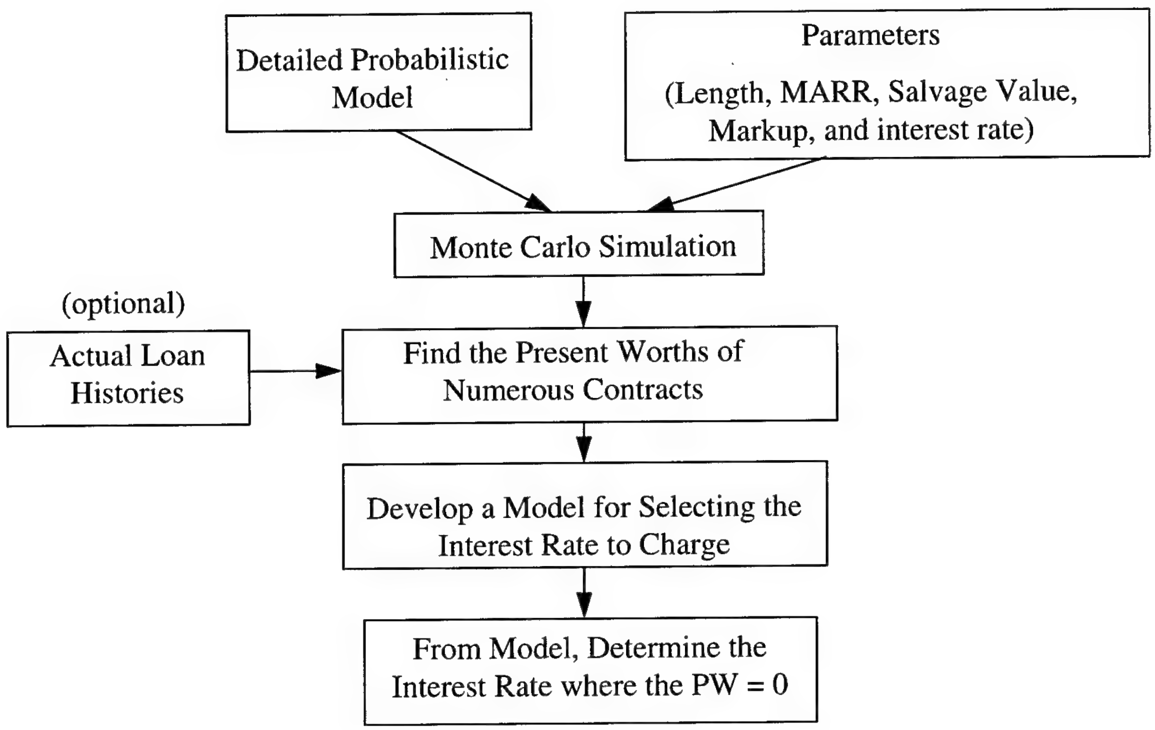


Figure 4-1 Logic Flow for Experiment

4.1 Design of Experiment

In designing this experiment, the central composite design (CCD) procedure was utilized. This method was used because there is not as much concern about the particular levels of each of the parameters of the model as there is about the response surface for the *PW*s the parameters create. The

CCD is a very efficient design in situations that call for a nonsequential batch response surface experiment [Myers 95]. For any given CCD design there are two parameters: α, the axial distance, and n_c, the number of center runs. Initially, there were four different parameters in the model: the face interest rate charged, the length of the contract, the markup on the furniture, and finally, the salvage value of the furniture. n_c, was chosen to be 3 and α equaled 2 from the equation below [Myers 95]:

$$\alpha = (2^k)^{1/4}$$

where k equaled 4, the number of parameters in the model. In order for a regression model to be good over a certain region, the boundaries for each of the parameters have to be set. These boundaries and the area of interest are shown in Table 3.

Table 4-1 Area of Interest

CCD	−2	−1	0	1	2
x1 (monthly f)	.0125	.01375	.015	.01625	.0175
x2 (length)	12	18	24	30	36
x3 (sal val)	.30	.40	.50	.60	.70
x4 (markup)	.40	.45	.50	.55	.60

middle of region
of interest

As expressed in the table above, x1 is the monthly face interest rate to be charged. These numbers correspond to nominal annual rates of .15, .165, .18, .185, and .21. The CCD design then becomes as shown in Table 4-2 on the following page

4.2 Calculation of the Expected Present Worth Using the Detailed Model

For each of the 27 points listed in Table 4-2, the FORTRAN program in Appendix B simulated 10,000 contracts using the probability matrices in Figure 3-4 and calculated an average present worth. Initial analysis indicated that perhaps a fifth predictor variable, the MARR, should be added to the model in order to increase the predictability of this model. The CCD design for four parameters was not redone to account for the fifth parameter. Instead, the 27 runs from Table 4-2 were executed for each of the MARRs in the area of interest. This area of interest varied from annual nominal rates of 12% to 21%.

Table 4-2 Central Composite Design

	X1 Interest Rate	X2 Length of Loan	X3 Salvage Value	X4 Markup
center pts	.015	24	.5	.5
	.015	24	.5	.5
	.015	24	.5	.5
axial pts	.0125	24	.5	.5
	.0175	24	.5	.5
	.015	36	.5	.5
	.015	12	.5	.5
	.015	24	.7	.5
	.015	24	.3	.5
	.015	24	.5	.6
	.015	24	.5	.4
factorial pts	.01625	30	.6	.55
	.01375	30	.6	.55
	.01625	18	.6	.55
	.01625	30	.4	.55
	.01625	30	.6	.45
	.01375	18	.6	.55
	.01375	30	.4	.55
	.01375	30	.6	.45
	.01625	18	.4	.55
	.01625	18	.6	.45
	.01625	30	.4	.45
	.01375	18	.4	.55
	.01375	18	.6	.45
	.01375	30	.4	.45
	.01625	18	.4	.45
	.01375	18	.4	.45

Therefore, the runs in Table 4-2 were performed at a MARR of 12%, 15%, 18%, and 21%, for a total of 108 runs. A CCD for five parameters would only give 45

different runs. Thus, by not redoing the CCD for five parameters, there was 63 more data points. As long as a fair amount of spread exists among the data points, more data points gives smaller variances of the estimated regression coefficients in the model. The CCD ensures this spread and thus lower variances should result.

The results for the first 27 runs with a MARR of 12% are shown in Table 4-3. The loan amount was $1200 in all cases. The annual and monthly interest rates are nominal.

4.3 Determination of a Model for the Interest Rate to be Charged

As shown in Figure 4-1, after obtaining the average present worths from the simulation runs, the goal was to determine a model for selecting the interest rate to charge, given the following five parameters: MARR, present worth, the length of the contract, and the salvage value and markup of the furniture. With this model a lender could select the best interest rate f to charge that would give a present worth of zero at MARR. Under normal CCD procedures, the response surface just created, for present worth in this experiment, would be the variable for which a regression equation is formed. Since the objective of this experiment is to find f, the regression equation is formed for f, using the present worth values that were just determined from the CCD. Using the Minitab statistical software package, the Best Subsets routine was utilized to determine the best fit model.

This routine explored best fits for different combinations of the five parameters with their higher orders and their higher order interactions.

Table 4-3 Experimental Results

Run	Pay-ment	annual f	monthly f	Length	Sal Val %	Markup %	monthly MARR	PW
1	59.91	0.18	0.015	24	0.5	0.5	0.01	5.56
2	59.91	0.18	0.015	24	0.5	0.5	0.01	4.02
3	59.91	0.18	0.015	24	0.5	0.5	0.01	5.09
4	58.18	0.15	0.0125	24	0.5	0.5	0.01	-32.89
5	61.66	0.21	0.0175	24	0.5	0.5	0.01	40.39
6	43.38	0.18	0.015	36	0.5	0.5	0.01	43.38
7	110.02	0.18	0.015	12	0.5	0.5	0.01	8.8
8	59.91	0.18	0.015	24	0.7	0.5	0.01	4.92
9	59.91	0.18	0.015	24	0.3	0.5	0.01	1.31
10	59.91	0.18	0.015	24	0.5	0.6	0.01	3.2
11	59.91	0.18	0.015	24	0.5	0.4	0.01	0.95
12	50.86	0.195	0.01625	30	0.6	0.55	0.01	23.65
13	49.09	0.165	0.01375	30	0.6	0.55	0.01	-15.45
14	77.43	0.195	0.01625	18	0.6	0.55	0.01	20.54
15	50.86	0.195	0.01625	30	0.4	0.55	0.01	20.47
16	50.86	0.195	0.01625	30	0.6	0.45	0.01	27.88
17	75.72	0.165	0.01375	18	0.6	0.55	0.01	-9.2
18	49.09	0.165	0.01375	30	0.4	0.55	0.01	-19.96
19	49.09	0.165	0.01375	30	0.6	0.45	0.01	-15.09
20	77.43	0.195	0.01625	18	0.4	0.55	0.01	17.74
21	77.43	0.195	0.01625	18	0.6	0.45	0.01	17.61
22	50.86	0.195	0.01625	30	0.4	0.45	0.01	26.18
23	75.72	0.165	0.01375	18	0.4	0.55	0.01	-12.45
24	75.72	0.165	0.01375	18	0.6	0.45	0.01	-10.1
25	49.09	0.165	0.01375	30	0.4	0.45	0.01	-18.47
26	77.43	0.195	0.01625	18	0.4	0.45	0.01	19.96
27	75.72	0.165	0.01375	18	0.4	0.45	0.01	-7.08

The factors in deciding which model to select from the set of Best Subsets were the following: a high R^2 (adj) with as few variables as possible; a small C_p value that corresponds to the number of variables in the model; and finally, low variance inflation factors (VIF); low is considered below 10 [Neter, 96]. R^2 measures the proportionate reduction of total variation in f with the use of a particular set of variables [Neter 96]. R^2 (adj) adjusts R^2 by dividing each sum of squares by its associated degrees of freedom; adding more variables to the model won't necessarily then increase the R^2 (adj) [Neter 96]. C_p is an estimator of the total mean squared error [Neter 96]. The VIFs measure how much the variances of the estimated regression coefficients are inflated as compared to when the predictor variables are not linearly related. The best model came out to be the three variable model which included the length of the contract, the present worth, and an interaction term consisting of the MARR and the length. This model can be seen in the regression analysis in Figure 4-2 on the following page.

Before proceeding further with the regression analysis, the normality assumptions of the linear model were verified using the residual model diagnostics program in Minitab. The data was then checked for outliers. Since no studentized deleted residuals exceeded the Bonferoni critical value, which for this experiment equaled 3.373, no f values were considered to be outliers [Neter, 96]. Several of the X value data points were found to be outlying by looking at the hat matrix for the data. This was because several exceeded the rule of

thumb value, defined to be equal to $2(p)/n$, where p is the number of parameters to include the intercept term in the model and n is the number of points [Neter, 96]. To see if these outlying points were influential in the model, the DFFITS and Cook's Distance were calculated for each; all points barely exceeded the justifiable values for DFFITS and Cook's Distance and thus were assumed not to be influential [Neter, 96]. The three variable model was:

$$f = .0149 - (0.000388 \times \text{length}) + (0.00007 \times PW) + (0.0385 \times \text{MARR} \times \text{length})$$

The regression equation is

f = 0.0149 -0.000387 length +0.000070 PW + 0.0384 MAR-len

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	0.0148637	0.0001162	128.08	0.000	
length	-0.00038730	0.00001145	-33.82	0.000	5.9
PW	0.00006978	0.00000162	42.09	0.000	6.8
MAR-len	0.0384051	0.0009713	39.54	0.000	14.4

s = 0.0002776 R-sq = 94.7% R-sq(adj) = 94.5%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	3	0.000142044	0.000047348	618.92	0.000
Error	104	0.000007956	0.000000077		
Total	107	0.000150000			

SOURCE	DF	SEQ SS
length	1	0.000000000
PW	1	0.000022451
MAR-len	1	0.000119593

Figure 4-2 Regression Analysis for the Three Variable Model

where again f is the monthly interest rate that should be charged for a given contract length, minimum monthly attractive rate of return, and present worth. This equation makes intuitive sense in that the higher the present worth, the higher f that must be charged to obtain that present worth. The interaction term between the MARR and length can be explained as follows. The higher the MARR, the higher the f needed to charge the borrower in order to obtain that MARR. The longer the length of the contract, the greater the chance of a default during the life of the contract. Thus the longer the length, the higher the f needed to be charged to compensate for that higher risk. The result is the positive interaction term. The negative coefficient on the length term serves as a damper on the effect of the length in the interaction term. Notice how this coefficient is much smaller than the interaction term.

In order to determine the interest rate that should be charged to break even with a present worth of zero, the present worth variable in the above equation is set to zero. Thus, the equation for selecting the interest rate f to charge a borrower based on a contract length and MARR is:

$$f = .0149 - (0.000388 \times \text{length}) + (0.0385 \times \text{MARR} \times \text{length})$$

4.3.1 Evaluation of the Estimation Model

The regression analysis output from Minitab for the three variable model was shown above in Figure 4-2. The adjusted coefficient of multiple

determination, denoted by R^2(adj), is very high with a value of 94.6%. This value would be much lower if 1,080,000 present worths were used in the model instead of the 108 present worths, each of which were an average for 10,000 contracts. The VIFs are acceptable; a value over 10 indicates that multicollinearity may exist for that variable [Neter, 96]. The interaction term in Figure 4-2 has a VIF of 14.4, but since this is an interaction term, multicollinearity problems would be expected. Ideally, the CCD should have resulted in VIFs that were zero because of the orthogonality of the design. This probably did not happen because the design was formed to regress against PW, not *f*, as was done in this experiment.

The standard deviation for this model is 0.0002776, which means that the monthly interest rate that was selected from a given set of parameters can be expected to vary by 0.02776% or annually by 0.33%. The test for whether or not there is a regression relation between the response variable *f* and the set of variables is the F test. For this test, the following hypothesis is tested:

$$H_0: \beta_1 = \beta_2 = \beta_3 = \ldots = \beta_{p-1}$$

$$H_1: \text{not all } \beta_k \ (k=1,\ldots,p-1) \text{ equal zero}$$

As can be seen by the large F-statistic of 618.92 calculated in Figure 4-2 by Minitab, the null hypothesis will be rejected in favor of the alternate hypothesis. Thus there appears to be a regression relationship between this set of variables

and f. *As* noted by the p-value, this rejection would occur at the 99.999% confidence level.

As shown by the regression equation, the salvage value and markup of the furniture are not significant in selecting f. If one wanted a model with more predictive power or with a higher R^2(adj), he might choose the 8 variable model shown in Figure 4-3 on the following page. As can be seen, with the addition of five variables this model has an R^2(adj) of 97.1. Once again, if a lender wanted to determine an interest rate to charge, he would utilize the regression equation in Figure 4-3 substituting in all applicable values to include a present worth of zero.

A purpose of the CCD is to keep the coefficients of quadratic terms from being negatively impacted by higher order terms. Since the resulting regression equation had no quadratic terms, a simple five factor factorial design could have been used instead of the CCD. This factorial design would have resulted in less simulation but should not have produced different results.

4.3.2 Validation of the Estimation Model

In order to validate this model, 10 other data points were found and applied to this model. The mean squared prediction error (MSPR) was calculated to be 7.7838E–7. This is a little greater than the mean squared error (MSE) for the model of 7.935E–6, showing that the predictive ability of this model is good and that the MSE for the selected regression model is not seriously biased [Neter 96].

The regression equation is

f = 0.00724 + 0.537 MARR(mo) +0.000108 pw -0.000004 len**2 + 0.0171 MAR-len -0.000000 pw**2 -0.000002 pw-len - 0.00136 sal-mrk +0.000063 len-mrk

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	0.0072385	0.0002954	24.50	0.000	
MARR(mo)	0.53684	0.03564	15.06	0.000	26.8
pw	0.00010818	0.00000340	31.86	0.000	57.2
len**2	-0.00000393	0.00000035	-11.15	0.000	25.2
MAR-len	0.017073	0.001455	11.73	0.000	61.7
pw**2	-0.00000004	0.00000001	-3.79	0.000	7.3
pw-len	-0.00000162	0.00000013	-12.93	0.000	63.8
sal-mrk	-0.0013611	0.0004047	-3.36	0.001	1.2
len-mrk	0.00006287	0.00001831	3.43	0.001	8.4

s = 0.0002001 R-sq = 97.4% R-sq(adj) = 97.1%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	8	0.000146036	0.000018255	455.96	0.000
Error	99	0.000003964	0.000000040		
Total	107	0.000150000			

SOURCE	DF	SEQ SS	Description
MARR(mo)	1	0.000000000	monthly nominal MARR
pw	1	0.000083263	present worth
len**2	1	0.000031532	length of contract squared
MAR-len	1	0.000024380	monthly nominal MARR × length
pw**2	1	0.000000003	present worth squared
pw-len	1	0.000006213	present worth × length of contract
sal-mrk	1	0.000000173	salvage value × markup on the furniture
len-mrk	1	0.000000472	length of the contract × markup

Figure 4-3 Regression Analysis for the Eight Variable Model

CHAPTER V

Conclusions and Recommendations

5.0 Overview

In this final chapter conclusions will be drawn about the Markov model and the simple regression model developed and then recommendations will be made concerning issues surrounding these models. Specifically, the first section of this chapter addresses issues concerning the two models, the Markov model and the regression model, in retail furniture loan pricing. It also discusses how the model measures against some evaluation criteria for regression models currently used in the literature. The next section summarizes the author's recommendations surrounding this research to improve the usefulness and accuracy of the regression model. Finally, the last section recommends future work that can be accomplished in retail furniture loan pricing.

5.1 Conclusions

In chapter II it was showed that there are few pricing techniques that incorporate risk in the lending industry. The goal in this research was to provide a simple model for the credit manager that could calculate an interest rate that would allow him to break even on a given loan that possesses risk. The rate at which the expected present worth for a risky loan is zero at the lender's risk-free

MARR is the break-even interest rate; this rate is the lowest rate the credit manager could charge and expect a profit, or at this rate, the manager can expect the same profit as for a risk-free loan at MARR.

5.1.1 Markov Model Formulation

The Markov process discussed in this thesis was a stand-in for whatever stochastic model would satisfactorily represent furniture-lending risk. Any such stochastic model would be based on conditional probabilities and would not be workable as a simple model for lenders actually to use in day-to-day operations. This version of the model was selected because it could use the only lending data that was available in the literature, which was for the mortgage industry. It was assumed that borrowers in the furniture and mortgage industry behave similarly. The mortgage-lending data was disaggregated somewhat so that it would be more representative of furniture-lending data. The results were simulated payment histories or furniture loans.

With respect to the Markov model, the following conclusions can be drawn. The state definitions have the disadvantage that the times of transitions from one state to another do not exactly determine the timings or numbers of payments. The states would have been able to give a better representation of a repayment schedule if instead of state 2 being 30-89 days late, it consisted of two different states, one for 30 days late and one for 60 days late. This would eliminate the necessity for the assumption that when a contract moved from states 3 to 2, there was a 50/50 chance of being one or two payments late.

Another improvement in the Markov model would be the following: instead of state 3 being 90+ days late, it would be better if there was a state for each additional month the payment was late over 90 days up until when the loan is characterized as defaulting. An example would be if a loan were characterized as being defaulted in its sixth month of being late, there would be six states instead of the one state used in the model. This would eliminate the necessity for another assumption: if a contract remained in state 3, no payment was made. With this improvement in the model, it would be possible for a contract to go from say 5 months late to three months late by making three payments, one for the current month, and two payments to catch up.

5.1.2 Regression (Simple) Model Formulation

As shown in the previous chapter, the final model which recommended to a credit manager is of the form:

$$f = .0149 - (0.000388 \times \text{length}) + (0.0385 \times \text{MARR} \times \text{length})$$

where f is the monthly nominal interest rate the credit manager should charge based on a given monthly nominal MARR and contract length. This regression model extracts the data very well from the collection of simulated payment histories as shown by the following statistics. The adjusted coefficient of multiple determination, denoted by R^2(adj), was very high with a value of 94.6%. Although a large value for R^2(adj) does not necessarily imply that the fitted model

is useful, the fact that it is high and the MSE is extremely small implies that inferences using this model may indeed be useful. Additionally, the standard deviation for this model was 0.0002776, which means that the monthly interest rate that is selected from a given set of parameters can be expected to vary by only 0.02776% or annually by 0.33%. Finally, the results showed that the smallest level of significance that would lead to the rejection of the null hypothesis (there was no regression relation between the response variable *f* and the set of variables) was a p value of 0.000. In other words, at any level of significance the null hypothesis would be rejected and it would be assumed that the set of variables has a regression relation with *f*.

The salvage value and markup of the furniture, which were originally thought to be significant parameters in the model, were not significant in selecting *f*.

An obvious advantage of this regression model is that it gives a simple formula for computing the interest rate a lender must charge in order for the expected present worth to be zero. A furniture credit manager can use this simple formula if he assumes that his borrowers behave as those do in the detailed probabilistic model. If he does not want to make this assumption, but would prefer to formulate his own simple model, he should follow the data collection process outlined in chapter III and also follow the same steps in finding the best equation.

A disadvantage of this model is that loans would have to be made at various parameters in order to create an appropriate representative response surface of present worths. It could be costly to make loans having yields below MARR, and difficult or illegal to make loans having yields above MARR. Another disadvantage of this simple model is that it is based only on information about the contract and not on information about the borrower. Obviously, certain qualities of the borrower will have an impact on the borrower's probability of defaulting. Such qualities as those mentioned by Caplovitz in chapter II and many others may result in significant predictor variables. These qualities can lead to subjective evaluations of the borrower; several of the models discussed in the second chapter were based on these subjective evaluations. This model is then applicable to those lending scenarios where lenders are prohibited from making subjective evaluations and can only use information pertaining to the contract for their decision as to which interest rate to charge.

Finally, since the data points used in this design were averages, this simple model selects the interest rate for the average borrower. If a prospective borrower is much different than the average borrower in that he has higher or lower transitional probabilities, the interest rate selected by this model may result in a much higher or lower present worth for the given contract.

5.1.3 Data Available

The greatest deficiency of this research was the lack of data pertaining to default probabilities for the furniture industry. As noted in chapters II and III, the only data available in the entire lending industry that pertained to default probabilities was for home mortgages. Data should be collected in the following manner: for each loan a record should be kept on the timing of each payment. If feasible, these records should be tracked by income, education, age, ethnic background, and location.

5.2 Recommendations

In order for more accurate research to be performed on this topic, data must be a collection of complete payment histories. These payment histories would be the amounts and timings of each repayment, exactly the same data that the simulation generated from the Markov model. Specifically then, the recommendations for data collection are as follows: 1) keep repayment records on actual loans as described in chapter III; 2) negotiate loans with longer and shorter lengths than the usual 24 month loan, and higher and lower markups, salvage values, and face interest rates (the MARR can be artificially varied since it influences repayment behavior only through the face interest rates and markup); 3) maintain a record for each of these loans of the income, education, age, ethnic background, and location of the borrower; 4) formulate a regression model in a similar fashion as performed in this thesis and then produce a simple formula; and finally 5) determine face interest rates using this formula.

5.3 Future Work

The first recommendation for further research on this topic is the appropriate data collection in the form of complete payment histories. Such data would assist the researcher in searching for additional predictor variables for the interest rate. This research began with five potential predictor variables, the length of the loan, the MARR, the salvage value and markup on the furniture, and the present worth of the contract, all characteristics of the loan and lender. Certainly, there are other predictors relating to the borrower, particularly those characteristics found by Caplovitiz. If data collection is not feasible, then perhaps the stochastic model developed in this thesis could be modified to account for these other predictor variables.

A final recommendation for future work pertains to the simple geometric model discussed in chapter III. It has been shown that with this model, a one parameter model can give the exact expected present worth as the model developed with 75 parameters. Further research might find a method of estimating this one parameter, q, which then can be used to find the interest rate f to be charged given the equation: $q = 1 - \frac{1+i}{1+f}$, where i is the MARR. While i and f have to do with the decision maker and his reaction to customer behavior respectively, q is a measure of customer behavior alone. Knowing q might have some utility in comparing situations for decision makers with varying i or MARR. The lender can easily determine what the face interest rate for any MARR being

considered should be so that the expected present worth will be zero. If capital was limited for the lender, he might choose to lend to a customer with a lower q.

Finally, since the geometric risk model can give the exact expected present worth of the detailed model, the credit manager could use the geometric risk model in determining the expected present worths of various types of contracts at different face interest rates. All he would have to is to estimate the single parameter q, and not have to analyze the many different profitability factors that may be involved.

APPENDIX A

DEPRECIATION FORTRAN PROGRAM

```
program geometric

*   Description:  The following program  calculates three different variables:
*   the balance on a loan after every payment has been made; the value
*   remaining on the furniture for every month; and finally, the loan/value
*   ratio which is the quotient of these two.  Double precision arithmetic
*   was used to ensure that the formulas being used were accurate.

*   Variables:

*    Type                   Name             Description
     double precision       dvalue(38)       ! The depreciated furniture value
     double precision       dq               ! The depreciation rate
     double precision       dbalance(38)     ! The balance on the loan
     double precision       dlv(38)          ! Loan divided by value
     double precision       dB               ! Used in PW calculation
     double precision       di               ! Face interest rate charged
     double precision       dA               ! Monthly payment
     double precision       dlength          ! Length of the loan
     double precision       dloan            ! Amount of the loan
     double precision       dr               ! Double precision of one
     double precision       dsalval          ! Salvage value of the furniture
     double precision d     markup           ! Markup on the furniture

     integer                k                ! Used as a counter in a do loop
     integer                t                ! Used as a counter

*    The following are the input parameters.

     dloan = 1200.0d0
     dmarkup = dloan/1.50d0
     dsalval = .5d0*dmarkup
```

```
      dlength = 24.0d0
      di = .18D0

*     Calculate the depreciation rate.

      dq = abs((dsalval/dmarkup)**(1.0d0/dlength)-1.0d0)

      print*, 'The depreciation rate per month is ', dq
      print*

*     Determine the value of the furniture for every month assuming an
*     initial value which incorporates the markup on the furniture.

      do k = 2, months + 1
       dvalue(1) = dmarkup
       dvalue(k) = dvalue(k-1) - dq*dvalue(k-1)
      end do

*     Determine the monthly payment required and then the balance on the
*     loan after every payment is made.

      dB = 1.0d0/(1.0d0 +di/12.0d0)
      dA = dloan*di/12.0d0/(1.0d0-dB**dlength)
      dr = 1.0d0
      t=1

      do while (t .le. months)
       dbalance(t) = dA * (1.0d0-dB**(dlength+1.0d0 - dr))
     A /(di/12.0d0)
       dlv(t) = dbalance(t)/dvalue(t+1)
       t = t+1
       dr = 1.0d0 + dr
      end do

      print*, 'The following represents the amortization and'
      print*, 'depreciation schedules for a', dlength, ' month loan'
      print*, 'at',dloan,' and at an interest rate of',di

      print*
      print*, 'The markup is ', dloan/dmarkup- 1.0d0, ' %.'
      print*, 'The salvage value is ', dsalval/dmarkup, ' %.'
      print*
      print*, 'Note that at time t, t-1 payments have occurred'
      print*
```

```
print*, '                  loan'
print*, '              balance at t  ','      value at t  ','
B  lv at t'
print*

do l=1, months
 print*, 't = ', l, dbalance(l), dvalue(l+1), dlv(l)
end do

print*
end
```

APPENDIX B

SIMULATION FORTRAN PROGRAM

```
program thesis

*     Description:  This program performs a simulation for thousands
*     of contracts.  This simulation creates payment histories given a
*     Markov model and given the following parameters for a contract:
*     length, markup, salvage value, face interest rate charged, & MARR.
*     The design of the experiment consists of 27 different sets of parameters
*     A given run will simulate 10,000 contracts for each of the 27 sets at
*     MARRs of 12%, 15%, 18%, and 21%.  The program then determines
*     the average present worth of these sets.

*     Variables:

*     Type    Name              Description

      real    prob1(5,5,3)      ! A list of the 75 stochastic probabilities
      real    I                 ! The MARR
      real    rv                ! A random variable
      real    rv2               ! A random variable
      real    rv3               ! A random variable
      real    epw(10000)        ! The expected present worth of one contract
      real    pw                ! A running total of the PW after each month
      real    value(40)         ! Value of the furniture as it depreciates
      real    q                 ! The depreciation rate
      real    sum               ! Used to sum the PW of the 10,000 contracts
      real    avgpw(27)         ! The avg pw of the 10,000 contracts
      real    B                 ! Equals 1/(1+ MARR)
      real    Annuity           ! What the borrower pays monthly
      real    balance(40)       ! The loan balance after a payment is made
      real    cussti(27)        ! Face interest rate charged to the customer
      real    bb                ! Equals 1/(1+cussti(z))
      real    salval(27)        ! The salvage value assumed on the furniture
```

```
        real      recovery(3)         ! % of the time the furniture is recoverable
        real      payment(3)          ! % moving from state 3 to state 2
        real      markup(27)          ! The markup on furniture

        integer   ISTRM               ! Used for random number generator
        integer   state               ! The state of the Markov process
        integer   matrix              ! Matrix the probability will be obtained from
        integer   length(27)          ! The length of the contract
        integer   number of reps      ! Contracts simulated for each set
        integer   o                   ! A counter in a do loop
        integer   paid early          ! Number of contracts paid off early
        integer   defaulted           ! Number of contracts defaulted
        integer   time 2              ! Amount of time spent in state 2
        integer   time 3              ! Amount of time spent in state 3
        integer   n                   ! A counter in a do loop
        integer   l                   ! A counter in a do loop
        integer   month               ! A counter in a do loop
        integer   rep                 ! A counter in a do loop
        integer   p                   ! A counter in a do loop
        integer   s                   ! A counter in a do loop
        integer   z                   ! A counter in a do loop
        integer   j                   ! A counter in an implied do loop
        integer   k                   ! A counter in an implied do loop
        integer   m                   ! A counter in an implied do loop
        integer   r                   ! A counter in an implied do loop
        integer   u                   ! A counter in an implied do loop
        integer   d                   ! A counter in an implied do loop
        integer   dd                  ! A counter in an implied do loop
        integer   ddd                 ! A counter in an implied do loop
        integer   dddd                ! A counter in an implied do loop

*       Functions
        real      RAND                ! Generates a random number

        number of reps = 10000

        istrm =1

*       This is how the data was read into the program for each of the 27 sets
*       of parameters.  The markup is based off a $1200 loan.  What is actually read
*       in here for markup is what the value of the furniture is for a given % markup.
*       The salvage value is the actual value of the furniture at the time of collection
*       based on a given % of the value of the furniture at the time of purchase, or *
*       the value read in for markup.
```

```
      data(length(d), d=1,27)/24,24,24,24,24,36,12,24,24,24,24,30,30,
     1  18,30,30,18,30,30,18,18,30,18,18,30,18,18/

      data(cussti(dd), dd=1,27)/.015,.015,.015,
     2 .0125,.0175,.015,.015,.015,.015,.015,.015,.01625,.01375,.01625,
     3 .01625,.01625,.01375,.01375,.01375,.01625,.01625,.01625,.01375,
     3 .01375,.01375,.01625,.01375/

      data(salval(ddd), ddd=1,27)/400,400,400,400,400,400,400,560,240
     4 ,375,428.57,464.52,464.52,464.52,309.68,496.55,464.52,309.68,
     5 496.55,309.68,496.55,331.03,309.68,496.55,331.03,331.03,331.03/

      data(markup(dddd), dddd=1,27)/800,800,800,800,800,800,800,800,
     6 800,750,857.14,774.19,774.19,774.19,774.19,827.59,774.19,
     7 774.19,827.59,774.19,827.59,827.59,774.19,827.59,827.59,
     8 827.59,827.59/

*     These are the different rates looked at.  The second of  each were the only
*      rates looked at.

      data(recovery(r), r = 1,3)/1.0,.5,0.0/
      data(payment(u), u = 1,3)/1.0,.5,0.0/

*     The 75 different probabilities that constitute the stochastic process.

      data (((prob1(j,k,m), k=1,5), j=1,5), m=1,3)/.979,.9985,.9985,
     a .9985,1,.715,.815,.915,.999,1,.32,.52,.82,1,1,0,0,0,1,1,0,0,
     b 0,0,1,.981,.999,.999,.999,1,.726,.826,.916,.999,1,.35,.54
     c ,.83,1,1,0,0,0,1,1,0,0,0,0,1,.979,.9985,.9985,
     d .9985,1,.715,.815,.915,.999,1,.32,.52,.82,1,1,0,0,0,1,1,0,0,
     e 0,0,1/

*      Runs each simulation, which consists of 10,000 contracts, 27 times, once for
*      each of the set of parameters.

      do z = 1,27

      q = abs((salval(z)/markup(z))**(1.0/length(z))-1)
      print*, q

      bb = 1/(1+cussti(z))
      annuity = (1200*cussti(z))/(1-bb**length(z))
      print*, 'z = ',z,' the annuity is ', annuity
```

```
      i = .12/12.0   ! Converts to a monthly rate.
      b = 1/(1+i)

*     Determines the value of the furniture for each month during the contract.

      do l = 2,length(z) + 1
        value(1) = markup(z)
        value(l) = value (l-1) - q*value(l-1)
        print*, l, ' the values are ', value(l)
      end do

*     Determines the balance left on the loan after each payment.

      do o = 1, length(z)
        balance(o) = Annuity*(1-bb**(length(z) +1-o))/cussti(z)
        print*, 'the balances are ',balance(o)
      end do

       p = 2   !recovery %
       do s = 2,2  !payments %
        sum = 0
        paid early = 0
        defaulted = 0

     do rep=1,number of reps  ! Simulates 10,000 contracts
        print*
        print*, 'contract ', rep
        pw = 0
        time2 = 0
        time 3 = 0
        state = 1

*  The following determines which matrix to obtain the transition probabilities
*  based on the month.  Note that there are three different matrices, one for the
*  first fourth of the number of months in the contract, one for the middle half
*  and one for the final fourth.

      do month = 1,length(z)

       if (length(z) .eq. 24) then
         if (month .le. 6) then
           matrix = 1
         else if (month .le. 18) then
```

```
  matrix = 2
 else
  matrix = 3
 end if

else if (length(z) .eq.18) then
 if (month.le.4)then
  matrix = 1
 else if (month.le.14) then
  matrix = 2
 else
  matrix = 3
 end if

else if (length(z) .eq.12) then
 if(month.le.3) then
  matrix = 1
 else if(month.le.9) then
  matrix = 2
 else
  matrix = 3
 end if

else if (length(z) .eq. 30) then
 if (month .le.7) then
  matrix = 1
 else if (month .le. 23) then
  matrix  = 2
 else
  matrix = 3
 end if

else if (length(z) .eq.36) then
 if (month.le.8) then
  matrix = 1
 else if (month.le.28) then
  matrix = 2
 else
  matrix = 3
 end if

end if
```

* Generate a random number and see where the transition goes.

```
      rv = rand(istrm)
      print*, 'rv for month ', month, rv

      if(rv .le. prob1(state,1,matrix)) then

        if(state .eq. 1)then

*    This was a transition from 1 to 1

          pw = annuity/(1+i)**month + pw

        else if (state .eq. 2) then

          if(time2 .eq. 1)then

*    A transition from 2 to 1

            pw = pw + 2*annuity/(1+i)**month
          else
            pw = pw + 3*annuity/(1+i)**month
          end if

        else if (state .eq. 3) then

*    A transition from 3 to 1

          pw = (time 3 + 3)*annuity/(1+i)**month

        end if

        state = 1
        time2 = 0

        print*, pw, ' after arriving in 1'
        go to 20

      else if (rv .gt. prob1(state,1,matrix) .and. rv .le.
     E    prob1(state,2,matrix)) then

        time 2  = time 2 + 1
        if (time 2 .gt. 2) then
          go to 50
        end if
```

```
          if (state .eq. 1 .or. state .eq. 2) then

*     A transition from 1 to 2

            pw = pw

          else if (state .eq. 3) then
            rv2 = rand(istrm)
            print*, 'rv2 is ', rv2
            if (rv2.le. payment(s)) then
              time2 = 2

*     A transition from 3 to 2

              pw = (time3 + 1)*annuity/(1+i)**month

            else
              pw = (time 3 + 2) *annuity/(1+i)**month
              time2 = 1
            end if

          end if
          state = 2
          time 3 = 0
          print*, pw, ' after arriving in 2'
          go to 20

        else if (rv .gt. prob1(state,2,matrix) .and. rv
     F     .le. prob1(state, 3,matrix))then
 50        pw = pw
          time2 = 0
          state = 3
          time 3 = time 3 +1

          print*, pw, ' after arriving in 3'
          go to 20

*     The following is if a contract defaults.

        else if (rv .gt. prob1(state,3,matrix) .and. rv
     G  .le. prob1(state,4,matrix)) then
          rv3 = rand(istrm)
          print*, 'rv3 is', rv3
```

```
        if (rv3 .le.recovery(p))then
          pw =  (value(month+1) - 400)/(1+i)**month
        else
          pw = pw - 200 * b**month
        end if

        defaulted = defaulted + 1

        state = 4
        print*, pw, ' pw for this contract after arriving in 4'
        go to 10

*   The following is if the contract moves to state 5 and is paid off.

      else if (rv .gt. prob1(state,4,matrix) .and. rv
   J      .le. 1) then
        pw = balance(month)*b**month + pw
        paid early = paid early + 1
        state = 5
        print*, pw, ' pw for this contract after arriving in 5'
        go to 10
      end if
20  end do

*   This maintains an array of pw for each of the 10,000 contracts.

10  epw(rep) = pw
    end do
!---------------------------------------------------------------------------------

*   Now add up this array of pw and determine an average.

    do n = 1,number of reps
      sum = sum + epw(n)
      print*, 'pw for contract',n, epw(n)
    end do

    avg pw(z) = (sum/(n-1))-1200
    print*
    print*, 'For a recovery rate of ', recovery (p),
   I ' and a payment rate of ', payment(s)

    print*, 'avg pw = ', avg pw(z), ' for a length of ', length(z),
```

```
   L' for an i of ',cussti(z), ' a salvage value of ', salval(z)
   M,' and a markup of ', markup(z)
    print*

    print*, 'the number of contracts that were paid early is ',
   H  paid early
    print*, 'the number that defaulted is ', defaulted
    print*, 'full term contracts ', number of reps - defaulted -
   J  paid early

    end do

    print*, '-------------------------------------------------------------'

    end do

    end

*   The following is a function which generates a random number.
*   It was developed by Marse and Roberts and modified and tested
*   by Law and Kelton. Please see further comments in Chapter III.

    Real function rand(istrm)

    Integer b2e15,b2e16,hi15,hi31,istrm,izset,low15,lowprd,
   &       modlus,mult1,mult2,ovflow,zi,zrng(100)
    integer irandg,randst

*    Force saving of ZRNG between calls.

    Save zrng

*    define the constants.

    data mult1,mult2/24112,26143/
    data b2e15,b2e16,modlus/32768,65536,2147483647/

*    Set the default seeds for all 100 streams.

    DATA ZRNG/1973272912, 281629770,
20006270,1280689831,2096730329,
   &        1933576050, 913566091, 246780520,1363774876, 604901985,
   &        1511192140,1259851944, 824064364, 150493284, 242708531,
   &          75253171,1964472944,1202299975, 233217322,1911216000,
```

```
&        726370533, 403498145, 993232223,1103205531, 762430696,
&       1922803170,1385516923,  76271663, 413682397, 726466604,
&        336157058,1432650381,1120463904, 595778810, 877722890,
&       1046574445,  68911991,2088367019, 748545416, 622401386,
&       2122378830, 640690903,1774806513,2132545692,2079249579,
&         78130110, 852776735,1187867272,1351423507,1645973084,
&       1997049139, 922510944,2045512870, 898585771, 243649545,
&       1004818771, 773686062, 403188473, 372279877,1901633463,
&        498067494,2087759558, 493157915, 597104727,1530940798,
&       1814496276, 536444882,1663153658, 855503735,  67784357,
&       1432404475, 619691088, 119025595, 880802310, 176192644,
&       1116780070, 277854671,1366580350,1142483975,2026948561,
&       1053920743, 786262391,1792203830,1494667770,1923011392,
&       1433700034,1244184613,1147297105, 539712780,1545929719,
&        190641742,1645390429, 264907697, 620389253,1502074852,
&        927711160, 364849192,2049576050, 638580085, 547070247/

*    Generate the next random number.

      Zi     = zrng(istrm)
      hi15   = zi / b2e16
      lowprd = (zi - hi15 * b2e16) * mult1
      low15  = lowprd / b2e16
      hi31   = hi15 * mult1 + low15
      ovflow = hi31 / b2e15
      zi     = (((lowprd - low15 * b2e16) - modlus) +
     &         (hi31 - ovflow * b2e15) * b2e16) + ovflow
      if (zi .lt. 0) zi = zi + modlus
      hi15   = zi / b2e16
      lowprd = (zi - hi15 * b2e16) * mult2
      low15  = lowprd / b2e16
      hi31   = hi15 * mult2 + low15
      ovflow = hi31 / b2e15
      zi     = (((lowprd - low15 * b2e16) - modlus) +
     &         (hi31 - ovflow * b2e15) * b2e16) + ovflow
      if (zi .lt. 0) zi = zi + modlus
      zrng(istrm) = zi
      rand   = (2 * (zi / 256) + 1) / 16777216.0
      return

*    Set the current ZRNG for stream ISTRM to IZSET.

      Entry randst(izset,istrm)
      zrng(istrm) = izset
```

```
      return

*     Return the current zrng for stream istrm.

      entry irandg(istrm)
      irandg = zrng(istrm)
      return

      END
```

BIBLIOGRAPHY

[Aguais 95] Aguais, Scott D., "Risk Systems Can Help Banks Survive the Slumps," *American Banker*, October 4, 1995

[Alexander 89] Alexander, Walter, "What's the Score?, " *ABA Banking Journal*, July 1989

[Anonymous 92] Anonymous, "Lots of Ways to Make Credit Decisions," *United States Banker*, February 1992

[Asarnow 95] Asarnow, Elliot, and David Edwards, "Measuring Loss on Defaulted Loans: A 24-Year Study," *The Journal of Commercial Lending,* March 1996

[Caplovitz 74] Caplovitz, David, "*Consumers in Trouble, A Study of Debtors in Default,*" *The Free Press*, 1974

[Casey 94] Casey, Mike K. and Ross N. Dickens, "Lenders: Treat Green Issues Cautiously," *Real Estate Review*, Summer 1994

[Chesser 74] Chesser, Delton L., "Predicting Loan Noncompliance," *The Journal of Commercial Lending*, August 1974

[Chmura 95] Chmura, Christine, "A Loan Pricing Case Study," *The Journal of Commercial Lending*, December 1995.

[Finn 92] Finn, W. Timothy II, and Joseph B. Frederick, "Managing the Margin," *ABA Banking Journal*, April 1992

[Gardner 89] Gardner, Mona J., and Dixie L. Mills, "Evaluating the Likelihood of Default on Delinquent Loans," *Financial Management*, Winter 1989

[Goodwin 93] Goodwin, William, "Loan Pricing Corporation Provides Banks with Information About Pricing," *American Banker*, May 12, 1993

[Haley 75] Haley, John, C., "Pricing Commercial Loans Today," *Banking*, March 1975

[Hally 96] Hally, Simon, "The Big Issue of Little Loans," *Canadian Banker*, January / February 1996

[Hendricks 77] Hendricks, James A., "Analysis of Risk in Capital Budgeting," *Management Accounting*, April 1977

[Johnson 90] Johnson, Richard, "The Impact of Deposit Balances on Loan Pricing and Profitability Analysis," *Banker's Magazine*, September 1990

[Koonce 96] Koonce, Ben, Telephonic Interviews on October 16, November 14, 1996, and February 5, 1997

[Kozlowski 96] Kozlowski, Jon, "Are You Getting What's Coming to You?" *ABA Banking Journal*, March 1996

[Lajaunie 95] Lajaunie, John, Thomas Stanley, and Craig Roger, "Loan Pricing," *The Banker's Magazine*, September-October 1995

[Laudeman 93] Laudeman, Mark, "Risk Rating Systems Bring Consistency to Commercial Lending," *Commercial Lending Review*, Summer 1993

[Law, 91} Law, Averill M. and David W. Kelton, *Simulation Modeling & Analysis*, McGraw-Hill, Inc., 1991

[Lawrence 96] Lawrence, Edward C., L. Douglas Smith, and Susan M. Sanchez, "Managing Credit Risk in a Changing Economy, " *The Banker's Magazine*, January 1996

[Lawrence 96] Lawrence, Edward C., L. Douglas Smith, and Susan M. Sanchez, "A Comprehensive Model for Managing Credit Risk on Home Mortgage Portfolios," *Decision Sciences*, Spring 1996

[Lesso 77] Lesso W.G., C.D. Zinn, and B. Motazed, "A Probabilistic Approach to Risk Analysis in Capital Investment Projects," *Engineering Economist*, Summer 1977

[Marshall 92] Marshall, Jeffrey, *Zap* – "The Computer Lops Off Another Bad Risk," *United States Banker*, August 1992

[Myers 95] Myers, Raymond H., and Douglas C. Montgomery, *Response Surface Methods*, John Wiley and Sons, Inc.,1995

[Neter, 96] Neter, John, Michael H. Kutner, Christopher J. Nachtsheim, and William Waserman, *Applied Linear Statistical Models*, fourth edition, Times Mirror Higher Education Group, Inc., 1996

[Pentikainen 88] Pentikainen, T., "Theory of Risk and Some Applications," *Journal of Risk and Insurance*, March 1988

[Stat Snapshot 94] Statistical Snapshot, "Loan Risk Ratings, Classification Systems, and Loss Factors," *The Journal of Commercial Lending,* May 1994

[Yang 91] Yang, Gilbert Y., "Pricing for Profit," *Banker's Magazine*, September-October 1991

[Young 75] Young, Donovan and Luis E. Contreras, "Expected Present Worths of Cash Flows Under Uncertain Timing," *The Engineering Economist*, Summer 1975

[Young 93] Young, Donovan, *Modern Engineering Economy*, John Wiley and Sons, Inc.,1993